Shore School

2014

Year 12 Mid-Year Examination

Examination Number:

Set:

Mathematics Extension 1

General Instructions

- Reading time − 5 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11–14, show relevant mathematical reasoning and/or calculations
- Write your examination number on the front cover of each booklet
- If you do not attempt a question, submit a blank booklet marked with your examination number and "N/A" on the front cover

Total marks – 70

Section I Pages 2–5

10 marks

- Attempt questions 1–10
- Allow about 15 minutes for this section

Section II Pages 6–10

60 marks

- Attempt Questions 11–14
- Allow about 1 hour 45 minutes for this section

Note: Any time you have remaining should be spent revising your answers.

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

Section I

10 marks
Attempt Questions 1−10
Allow about 15 minutes for this section

Use the Multiple Choice Answer Sheet for Questions 1−10.

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1 Which expression is a correct factorisation of $27x^3+8$?

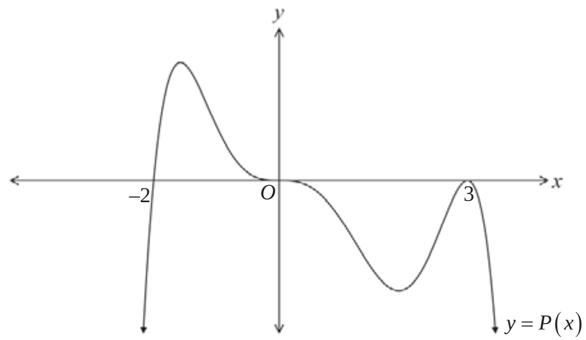
(A) $(3x+2)(9x^2-12x+4)$

(B) $(3x+2)(9x^2-6x+4)$

(C) $(3x+2)(9x^2+12x+4)$

(D) $(3x+2)(9x^2+6x+4)$

2 A polynomial is sketched on the axes below.



Which of the following could be the equation of the polynomial?

(A) $P(x)=-x^3(x+2)(x-3)^2$

(B) $P(x)=x(x+2)(x-3)$

(C) $P(x)=x^3(x+2)(x-3)^2$

(D) $P(x)=x^3(x-2)(x+3)^2$

3 What is $\lim_{x\to 0} \frac{\sin \frac{x}{4}}{2x}$?

(A) 0

(B) $\frac{1}{8}$

(C) $\frac{1}{2}$

(D) 8

4 What is the derivative of $\sin^{-1}(5x)$?

(A) $\frac{-5}{\sqrt{1-25x^2}}$

(B) $\frac{-1}{\sqrt{1-25x^2}}$

(C) $\frac{1}{\sqrt{1-25x^2}}$

(D) $\frac{5}{\sqrt{1-25x^2}}$

5 Given that $\sin\theta = \frac{2t}{1+t^2}$ and $\cos\theta = \frac{1-t^2}{1+t^2}$ when $t = \tan\frac{\theta}{2}$, which of the following is a simplified expression for $\sin\theta\sec\theta$?

(A) $\frac{2}{1-t}$

(B) $\frac{1-t}{2}$

(C) $\frac{1-t^2}{2t}$

(D) $\frac{2t}{1-t^2}$

6 Consider the function $f(x) = \frac{3x^4 + 5x^2}{x^4 + 5}$.
Which one of the following statements is correct?

(A) $f(x)$ is odd and $\lim_{x\to\infty} f(x) = 3$

(B) $f(x)$ is odd and $\lim_{x\to\infty} f(x) = 5$

(C) $f(x)$ is even and $\lim_{x\to\infty} f(x) = 3$

(D) $f(x)$ is even and $\lim_{x\to\infty} f(x) = 5$

7 The cubic polynomial $P(x)$ has roots α, β, and γ such that $\alpha+\beta+\gamma=5$ and $\alpha\beta+\alpha\gamma+\beta\gamma=4$
Which of the following could be the equation of $P(x)$?

(A) $P(x) = x^3 + 5x^2 - 4x - 20$

(B) $P(x) = x^4 - 5x^3 + 4x^2 - x + 11$

(C) $P(x) = 2x^3 - 5x^2 + 4x - 20$

(D) $P(x) = 2x^3 - 10x^2 + 8x + 17$

8 Which of the following gives all of the solutions to $\left(1-\frac{1}{x}\right)^2 + 3\left(1-\frac{1}{x}\right) - 4 = 0$?

(A) $x=-4,\ x=1$

(B) $x=4,\ x=-1$

(C) $x=\frac{1}{5}$ only

(D) $x=\frac{1}{5},\ x=0$

9 What is the derivative of $x\sec^2 x$?

(A) $\dfrac{\cos^2 x + x\sin 2x}{\cos^4 x}$

(B) $\dfrac{\cos^2 x - 2x\cos x}{\cos^4 x}$

(C) $\dfrac{-2x}{\cos^3 x}$

(D) $x\tan x + \sec^2 x$

10 It is known that $\tan A = 4$ and $\tan(A+B) = \dfrac{1}{2}$.

What is the value of $\tan B$?

(A) $-3\dfrac{1}{2}$

(B) $-\dfrac{7}{6}$

(C) -1.292

(D) -0.862

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet

(a) Solve the inequality $\dfrac{x+3}{x-2} > -1$. **3**

(b) Consider the function $f(x) = e^{5x+4}$. **2**
Find $f^{-1}(x)$, the inverse function of $f(x)$.

(c) Find $\displaystyle\int_0^1 \frac{dx}{\sqrt{4-x^2}}$, giving your answer in terms of π. **2**

(d) Using the substitution $u = 1+2x$, find $\displaystyle\int \frac{6\,dx}{\sqrt{(1+2x)^3}}$. **3**

(e) The acute angle between the lines $3x - y + 7 = 0$ and $mx - y + 1 = 0$ is 45°. **2**
Find the possible value(s) of m.

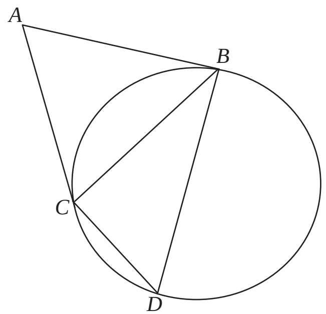
(f) When the polynomial $P(x) = x^3 + px^2 + qx - 4$ is divided by $(x-2)$ the remainder is 12. Also, $(x+1)$ is a factor of $P(x)$. **3**
Find the values of p and q.

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) (i) Express $3\cos x + \sqrt{3}\sin x$ in the form $R\cos(x-\alpha)$, where $0 < \alpha < \frac{\pi}{2}$ and $R > 0$. **2**

(ii) Hence, or otherwise, find the general solution of the equation $3\cos x + \sqrt{3}\sin x = \sqrt{6}$. **3**

(b) AB and AC are tangents to a circle. D is a point on the circle such that $2 \times \angle DBC = \angle BAC$ and $\angle BDC = \angle BAC$.



NOT TO SCALE

Copy or trace the diagram into your writing booklet.

(i) Show that DB is a diameter. **3**

(ii) Show that $BC = AB$. **1**

Question 12 continues on page 8

(c) Consider the function $f(x) = 4x - x^3$.

(i) Sketch $y = f(x)$, showing the x and y intercepts and the coordinates of the stationary points. **3**

(ii) Find the largest domain containing the origin for which $f(x)$ has an inverse function, $f^{-1}(x)$. **1**

(iii) State the domain of $f^{-1}(x)$. **1**

(iv) Find the gradient of the inverse function at the origin. **1**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) (i) Draw a neat graph of $y = 3\sin 2x$, for $-\pi \le x \le \pi$, indicating all intercepts on the x axis. **2**

(ii) Use your graph to determine the number of solutions to the equation $3\sin 2x = x$. **1**

(iii) If $m > 0$, for what values of m will the equation $3\sin 2x = mx$ have only one solution? **2**

(b) Prove by mathematical induction that for all integers $n \ge 2$, **3**

$$1\times 2+2\times 3+3\times 4+\dots\dots\dots+(n-1)\times n=\frac{1}{3}(n-1)n(n+1).$$

(c) Consider the function $f(x) = x\log_e x - 1$ with $x > 0$.

(i) Find the coordinates of the stationary point on $y = f(x)$ and determine its nature. **2**

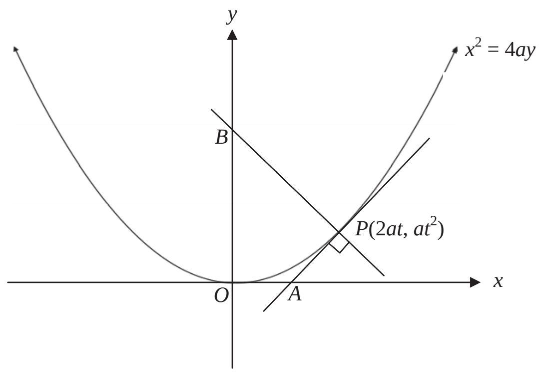
(ii) Let $x = 2$ be a first approximation to the root of the equation $x\log_e x - 1 = 0$. **2**
Use one application of Newton's method to approximate the x-intercept. Leave your answer correct to 2 decimal places.

(iii) Explain why the curve $y = f(x)$ is concave up for all $x > 0$. **1**

(iv) Sketch the curve, showing all its main features. **2**

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) The point $P(2at, at^2)$ lies on the parabola $x^2 = 4ay$.



NOT TO SCALE

(i) Show that the tangent to the parabola at P meets the x axis at $A(at, 0)$. **2**

(ii) Show that the normal to the parabola at P meets the y axis at $B(0, 2a + at^2)$. **2**

(iii) The point R divides BA **externally** in the ratio $2:1$.

(α) Show that the coordinates of R are $(2at, -2a - at^2)$. **1**

(β) Show that R lies on a parabola with the same directrix and focal length as the original parabola. **3**

(b) Consider the function $y = \frac{1}{2}\cos^{-1}(x-1)$.

(i) What is the domain and range of the function? **2**

(ii) Sketch the graph of the function showing the coordinates of the endpoints. **1**

(iii) The region in the first quadrant bounded by the curve $y = \frac{1}{2}\cos^{-1}(x-1)$ and the coordinate axes is rotated about the y-axis. Find the volume of the solid of revolution. Express your answer in simplest exact form. **4**

End of paper

Year 12 Mid-Year Extension 1 Maths Solutions

Section I

1. B
2. A
3. B

$$\lim_{x\to 0}\frac{\sin\frac{x}{4}}{2x} = \lim_{x\to 0}\frac{\sin\frac{x}{4}}{\frac{x}{4}}\cdot\frac{1}{8}$$
$$= 1\times\frac{1}{8}$$
$$= \frac{1}{8}.$$

4. D.

$$\frac{d\left(\sin^{-1}(5x)\right)}{dx} = \frac{1}{\sqrt{1-(5x)^2}}\cdot 5$$
$$= \frac{5}{\sqrt{1-25x^2}}$$

5. D.

$$\sin\theta\sec\theta = \sin\theta\cdot\frac{1}{\cos\theta}$$
$$= \frac{\sin\theta}{\cos\theta}$$
$$= \frac{\frac{2t}{1+t^2}}{\frac{1-t^2}{1+t^2}}$$
$$= \frac{2t}{1-t^2}$$

6. C.

$$\lim_{x\to\infty}\frac{3x^4+5x^2}{x^4+5} = \lim_{x\to\infty}\frac{3+\frac{5}{x^2}}{1+\frac{5}{x^4}}$$
$$= 3.$$

Section I (cont.)

7. D.

$$\alpha\beta+\alpha\gamma+\beta\gamma = \frac{c}{a}$$
$$= 4$$
$$\alpha+\beta+\gamma = \frac{-b}{a}$$
$$= 5.$$

8. C

Let $u = 1-\frac{1}{x}$

$$u^2+3u-4=0$$
$$(u+4)(u-1)=0$$
$$u=1 \quad \text{or} \quad u=-4.$$
$$\therefore 1-\frac{1}{x}=1 \quad \text{or} \quad 1-\frac{1}{x}=-4$$
$$-\frac{1}{x}=0 \qquad\qquad -\frac{1}{x}=-5$$

no solution $\qquad x=\frac{1}{5}.$

$\therefore x=\frac{1}{5}$ only.

9. A

$$\frac{d\,(x.\sec^2x)}{dx} = \cancel{1.\sec^2x+}$$
$$= \frac{d}{dx}\left(\frac{x}{\cos^2x}\right)$$
$$= \frac{1.\cos^2x - 2.\cos x\,x\,-\sin x.\,x}{(\cos^2x)^2}$$
$$= \frac{\cos^2x + x.\,2\sin x\cos x}{\cos^4x}$$
$$= \frac{\cos^2x + x.\sin 2x}{\cos^4x}.$$

Section I (cont.)

10. B. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$$\frac{1}{2} = \frac{4 + \tan B}{1 - 4\tan B}$$

$$1 - 4\tan B = 8 + 2\tan B$$

$$-7 = 6\tan B$$

$$\tan B = \frac{-7}{6}.$$

Section 2

Q11

a) $\dfrac{x+3}{x-2} > -1 \quad , \quad x \neq 2.$

Let $x + 3 = -(x-2)$

$$2x = \cancel{2\tfrac{1}{2}} - 1$$

$$x = -\frac{1}{2}$$

Test $x = 0$

$$\frac{0+3}{0-2} = -\frac{3}{2}$$

$$\not> -1$$

$\therefore$

$x > 2$ or $x < -\frac{1}{2}$.

OR

11a) $\dfrac{x+3}{x-2} > -1 \quad , \quad x \neq 2.$

$$(x+3)(x-2) > -(x-2)^2$$

$$x^2 + x - 6 > -x^2 + 4x - 4$$

$$2x^2 - 3x - 2 > 0$$

Let $2x^2 - 3x - 2 = 0$

$$(2x+1)(x-2) = 0$$

$$x = 2 \text{ or } x = -\frac{1}{2}.$$

Test $x = 0$. (as above).

$\therefore \quad x > 2$ or $x < -\frac{1}{2}$.

b) $f(x) = e^{5x+4}$

Let $y = e^{5x+4}$

$$\ln y = 5x + 4$$

$$x = \frac{\ln y - 4}{5}.$$

$$\therefore f^{-1}(x) = \frac{\ln x - 4}{5}.$$

c) $$\int_0^1 \frac{dx}{\sqrt{4 - x^2}} = \int_0^1 \frac{dx}{\sqrt{2^2 - x^2}}$$

$$= \left[\sin^{-1}\left(\frac{x}{2}\right)\right]_0^1$$

$$= \sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(0)$$

$$= \frac{\pi}{6}.$$

11 d) $\displaystyle\int \frac{6\,dx}{\sqrt{(1+2x)^3}}$

Let $u = 1+2x$

$\frac{du}{dx} = 2$

$du = 2\,dx$

$3du = 6\,dx$

$= \displaystyle\int \frac{3\,du}{\sqrt{u^3}}$

$= 3\displaystyle\int u^{-\frac{3}{2}}\,du$

$= 3\,\dfrac{u^{-\frac{1}{2}}}{-\frac{1}{2}} + C$

$= \dfrac{-6}{\sqrt{u}} + C$

$= \dfrac{-6}{\sqrt{1+2x}} + C.$

e) $\tan\alpha = \left|\dfrac{m_1 - m_2}{1+m_1m_2}\right|$ $\qquad m_1 = 3$, $m_2 = m$

$1 = \left|\dfrac{3-m}{1+3m}\right|$

$1+3m = 3-m$ or $1+3m = m-3$

$4m = 2$ or $2m = -4$

$m = \frac{1}{2}$ or $m = -2.$

Q11 f) $P(2) = 12$

$2^3 + p.2^2 + q.2 - 4 = 12$

$8 + 4p + 2q - 4 = 12$

$4p + 2q = 8$ ①.

$P(-1) = 0$

$(-1)^3 + p(-1)^2 + q(-1) - 4 = 0$

$p - q = 5.$ ②.

$p = 5+q$ ③

③ → ①: $4(5+q) + 2q = 8.$

$20 + 4q + 2q = 8$

$6q = -12$

$q = -2.$

$\therefore p = 3,\ q = -2.$

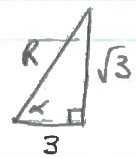
Q12

a) i) $R\cos(x-\alpha) = R\cos x\cos\alpha + R\sin\alpha\sin x$

So $3\cos x + \sqrt{3}\sin x = R\cos x\cos\alpha + R\sin x\sin\alpha$

Equating coefficients

$R\cos\alpha = 3$, $R\sin\alpha = \sqrt{3}.$



So $\tan\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \frac{\pi}{6}$ for $0 < \alpha < \frac{\pi}{2}$

$R^2 = 3^2 + (\sqrt{3})^2$

$R = \sqrt{12}$, $R > 0.$

$\therefore 3\cos x + \sqrt{3}\sin x = \sqrt{12}\cos\left(x - \frac{\pi}{6}\right).$

Q12a) ii)

$3\cos x + \sqrt{3}\sin x = \sqrt{6}$

So $\sqrt{12}\cos\left(x - \frac{\pi}{6}\right) = \sqrt{6}$

$\cos\left(x - \frac{\pi}{6}\right) = \sqrt{\frac{1}{2}}$

$\cos\left(x - \frac{\pi}{6}\right) = \frac{1}{\sqrt{2}}$

$x - \frac{\pi}{6} = \pm\frac{\pi}{4} + 2n\pi$, where n is an integer

$\therefore x = \frac{\pi}{6} + \frac{\pi}{4} + 2n\pi$ or $x = \frac{\pi}{6} - \frac{\pi}{4} + 2n\pi$

$= \frac{5\pi}{12} + 2n\pi$ or $x = \frac{-\pi}{12} + 2n\pi$,

$n \in \mathbb{Z}$

b) $\angle ABC = \angle BDC$ (alternate segment theorem)

$\angle ACB = \angle BDC$ (alternate segment theorem)

$\angle BAC = \angle BDC$ (given)

$\therefore \triangle ABC$ is equilateral and

$\angle ABC = \angle ACB = \angle BAC = \angle BDC = 60°$.

$\angle DBC = \angle BAC \div 2$ (given)

$= 30°$

Now $\angle ABD = \angle ABC + \angle DBC$

$= 60° + 30°$

$= 90°$.

$\therefore DB$ is a diameter

(tangent and diameter through point of contact meet at right angles).

Q12b) ii)

From (i), $\triangle BAC$ is equilateral.

$\therefore BC = AB$.

c) (i) $f(x) = 4x - x^3$

$= x(4 - x^2)$

$= x(2-x)(2+x)$

y-intercept when $x = 0$

$y = 0 - 0$

$= 0$

x-intercept when $y = 0$

$0 = x(2-x)(2+x)$

$x = 0, \pm 2$.

$f'(x) = 4 - 3x^2$

Stat. pts when $f'(x) = 0$

$0 = 4 - 3x^2$

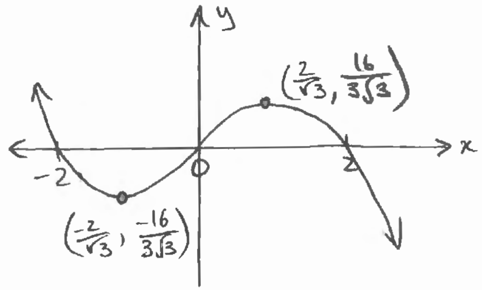
$x^2 = \frac{4}{3}$

$x = \pm\frac{2}{\sqrt{3}}$

When $x = \frac{\pm 2}{\sqrt{3}}$, $f(x) = 4\left(\pm\frac{2}{\sqrt{3}}\right) - \left(\pm\frac{2}{\sqrt{3}}\right)^3$

$= \frac{8}{\sqrt{3}} - \frac{8}{3\sqrt{3}}$ or $\frac{-8}{\sqrt{3}} + \frac{8}{3\sqrt{3}}$

$= \frac{16}{3\sqrt{3}}$ or $\frac{-16}{3\sqrt{3}}$



Q12 c) ii)

$f(x)$ is monotonic increasing for

$$\frac{-2}{\sqrt{3}} \le x \le \frac{2}{\sqrt{3}}.$$

$\therefore$ this is the largest domain containing the origin for which $f^{-1}(x)$ exists.

iii) Domain of $f^{-1}(x)$: $\frac{-16}{3\sqrt{3}} \le x \le \frac{16}{3\sqrt{3}}$

iv) $f'(x) = 4 - 3x^2$

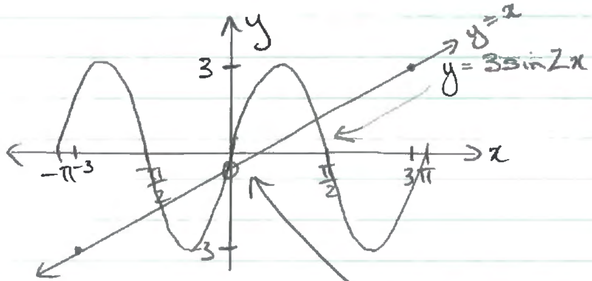
$f'(0) = 4$

$\therefore$ gradient of inverse function at origin is $\frac{1}{4}$.

Q13 a) i) Period $= \frac{2\pi}{2}$

$= \pi$

Amplitude $= 3$.



ii) see above

$\therefore$ 3 solutions.

Q13) a)

iii) $3\sin 2x = mx$ will have only one solution when

$$\frac{d(mx)}{dx} \geqslant \frac{d(3\sin 2x)}{dx}\Big|_{x=0}.$$

$$m \geqslant 6\cos 2x\,\Big|_{x=0}.$$

$$m \geqslant 6.$$

b) Prove true when $n=2$.

LHS $= 1\times 2$
$= 2$

RHS $= \frac{1}{3}\times 1\times 2\times 3$
$= \frac{1}{3}\times 6$
$= 2$

LHS = RHS

$\therefore$ true for $n=2$.

Assume true for $n=k$

i.e.

$$1\times 2 + 2\times 3 + 3\times 4 + \ldots + (k-1)k = \tfrac{1}{3}(k-1)k(k+1).$$

Prove true for $n=k+1$.

i.e. required to prove that

$$1\times 2 + 2\times 3 + 3\times 4 + \ldots + (k-1)k + k(k+1) = \tfrac{1}{3}k(k+1)(k+2).$$

LHS $= \frac{1}{3}(k-1)k(k+1) + k(k+1)$ by the inductive hypothesis

$= \frac{1}{3}k(k+1)[(k-1)+3]$

$= \frac{1}{3}k(k+1)(k+2)$

$=$ RHS.

Q13) b)

∴ by the principle of mathematical induction, statement is true for $n \geq 2$, $n \in \mathbb{Z}$.

c). i) $f(x) = x \ln x - 1$, $x > 0$

$f'(x) = 1.\ln x + \frac{1}{x}\cdot x - 0$

$= 1 + \ln x$

Stat. pts when $f'(x) = 0$

$0 = 1 + \ln x$

$\ln x = -1$

$x = e^{-1}$

$= \frac{1}{e}$.

when $x = \frac{1}{e}$, $f(x) = \frac{1}{e}\cdot -1 - 1$

$= \frac{-1}{e} - 1$

$f''(x) = \frac{1}{x}$

> 0 when $x > 0$.

∴ stationary point is a local minimum. (curve is concave up).

∴ the stationary point at $\left(\frac{1}{e}, \frac{-1}{e} - 1\right)$ is a minimum turning point.

Q13) c) ii)

$f(x) = x \ln x - 1$

$f'(x) = \ln x + 1$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 2 - \frac{2.\ln 2 - 1}{\ln 2 + 1}$

$= 1.771848327\ldots$

$\doteqdot 1.77$

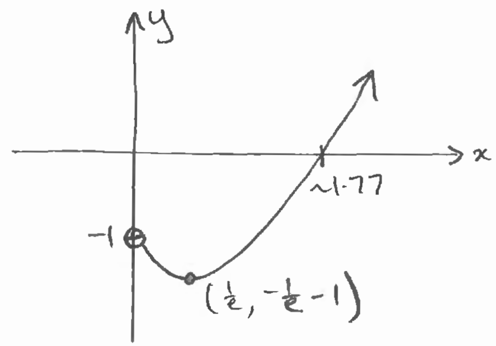
∴ the x-intercept is approximately $x \doteqdot 1.77$

iii) $f''(x) = \frac{1}{x}$ and $x > 0$

∴ $f''(x) > 0$ for $x > 0$

∴ $y = f(x)$ is concave up for $x > 0$.

iv)



$\lim_{x \to 0} x \ln x - 1 = -1$

Q14 a)

i) gradient of tangent at $P = t$

tangent at P: $y - at^2 = t(x - 2at)$

$y = tx - at^2$

When $y = 0$:

$0 = tx - at^2$

$at^2 = tx$

$x = at$

$\therefore A = (at, 0)$.

ii) gradient of normal at $P = \frac{-1}{t}$

normal at P: $y - at^2 = \frac{-1}{t}(x - 2at)$

$yt - at^3 = -x + 2at$

When $x = 0$:

$yt - at^3 = 2at$

$yt = 2at + at^3$

$y = 2a + at^2$

$\therefore B = (0, 2a + at^2)$.

iii) α) $A(at, 0)$ $\quad$ $B(0, 2a + at^2)$

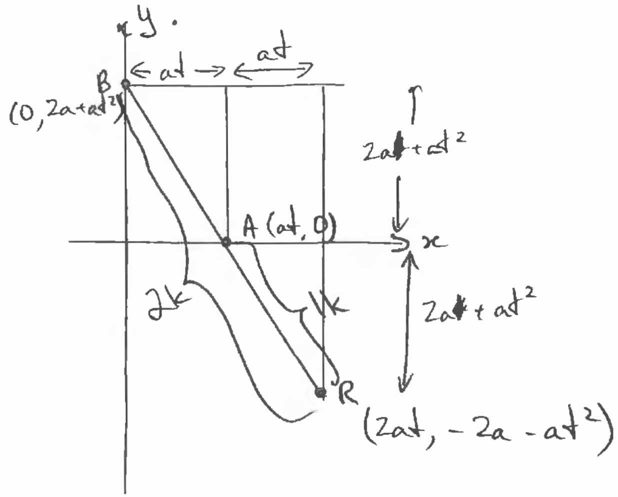
$-1 : 2$

$\therefore x = 2at,\ y = -2a - at^2$

$\therefore R = (2at, -2a - at^2)$.

OR

α) cont.



By similar triangles,

$R = (2at, -2a - at^2)$

β) $x = 2at,\quad y = -2a - at^2$

$t = \frac{x}{2a}$

$\therefore y = -2a - a\left(\frac{x}{2a}\right)^2$

$= -2a - \frac{ax^2}{4a^2}$

$= -2a - \frac{x^2}{4a}$

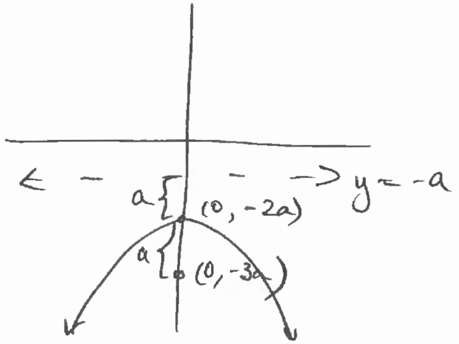
$\frac{x^2}{4a} = -y - 2a$

$= -(y + 2a)$

$x^2 = -4a(y + 2a)$

14 a) iii) β) cont.

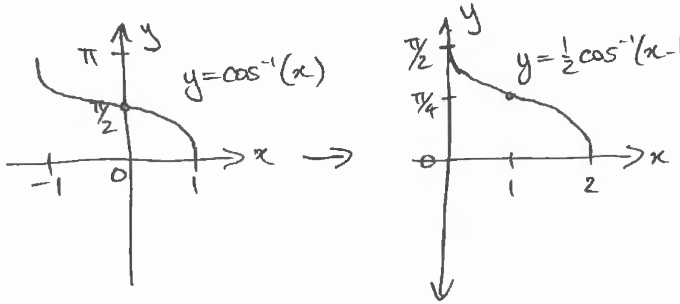
So ~~verte~~ this is a concave down parabola with focal length = a, vertex = (0, −2a)



∴ directrix is $y = -a$.

∴ focal length = a and directrix is $y = -a$, same as original parabola.

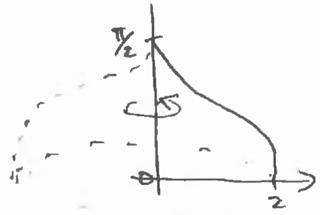
14) b) ii)



i) Domain: $0 \le x \le 2$

Range: $0 \le y \le \frac{\pi}{2}$

Q14) b) iii)



$$y = \tfrac{1}{2}\cos^{-1}(x-1)$$
$$2y = \cos^{-1}(x-1)$$
$$x - 1 = \cos 2y.$$
$$x = \cos 2y + 1$$

$$V = \pi \int_0^{\frac{\pi}{2}} x^2 \, dy.$$

$$= \pi \int_0^{\pi/2} (\cos 2y + 1)^2 \, dy$$

$$= \pi \int_0^{\pi/2} \cos^2 2y + 2\cos 2y + 1 \, dy.$$

$$= \pi \int_0^{\pi/2} \frac{\cos 4y + 1}{2} + 2\cos 2y + 1 \, dy.$$

$$= \pi \left[\frac{\sin 4y}{8} + \frac{3y}{2} + \sin 2y \right]_0^{\pi/2}$$

$$= \pi \left[\left(\cancel{\frac{\sin 2\pi}{8}}^{\,0} + \frac{3\pi}{4} + \cancel{\sin \pi}^{\,0} \right) - (0 + 0 + 0) \right]$$

$$= \pi \cdot \frac{3\pi}{4}$$

$$= \frac{3\pi^2}{4} \text{ units}^3$$